All-to-All Communication

Principles of Distributed Computing 2013

Thomas Locher

ETH Zurich – Distributed Computing – www.disco.ethz.ch

Overview

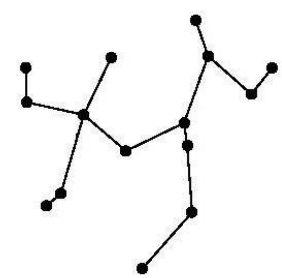
Overview

I. Introduction: Definitions

• A tree is a connected graph without cycles.

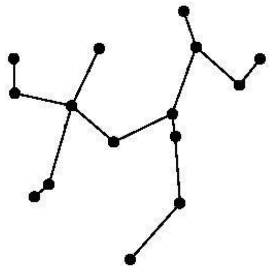
• A subgraph that spans all vertices of a graph is called a spanning subgraph.

• Among all the spanning trees of a weighted and connected graph, a spanning tree with the least total weight is called a minimum spanning tree (MST).



I. Introduction: Definitions

• In a MST algorithm, |V| - 1 edges have to be chosen in total. In each phase of the algorithm, probably only a fraction of those edges are chosen.



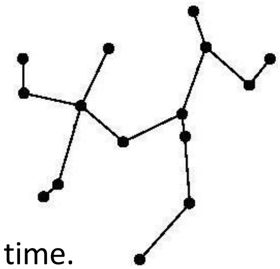
• Nodes that are directly or indirectly connected using chosen edges only belong to the same cluster.

•The minimum weight outgoing edge (MWOE) is the edge with the lowest weight among all incident edges leading to other clusters.

I. Introduction: Usage of MST

• Minimize the cost associated with global operations such as broadcast!

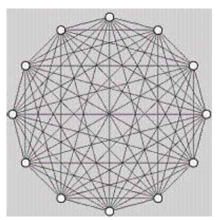
• Minimize the message complexity: Avoid traffic explosion by using a spanning tree (no cycles!)



• Minimize the time complexity: If the edge weights represent the delays on the links, then a MST minimizes the execution time.

I. Introduction: System Model

• The system is represented by a complete weighted undirected graph G=(V,E,w) where w(e) denotes the weight of edge $e \in E$ and |V|=n.



• All edge weights are different (w.l.o.g.).

• Each node has a distinct ID of O(log n) bits.

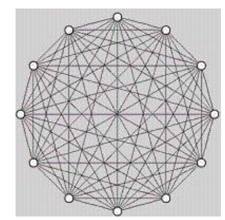
• Each node knows all the edges it is incident to and their weights.

• Each node knows about all the other nodes.

• The synchronous communication model is used.

I. Introduction: Synchronous Communication Model

• Communication advances in global rounds.



• In each round, processes send messages, receive messages, and do some local computation.

• The time complexity is the number of rounds until the computation terminates in the worst case.

• The message complexity is the number of messages/bits exchanged in the worst case.

I. Introduction: Getting a Feeling for the Problem...

- How hard is it to compute the MST in a distributed system (assuming a fully connected graph)?
- All nodes know the weights of all incident edges. If all nodes send this information to all other nodes, then all nodes suddenly have the entire picture.
- A simple algorithm that requires only one round!
- However, that is not really interesting...
- Therefore, the message size is limited to O(log n) bits!
- Note that the simple algorithm requires messages of size O(n log n)!

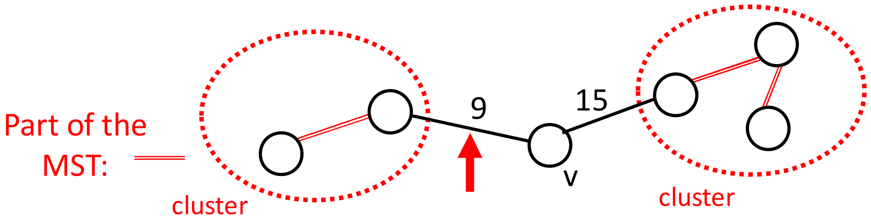
I. Introduction: Getting a Feeling for the Problem...

- Since each node ID (and edge weight) requires O(log n) bits, this implies that only a constant number of node IDs (and edge weight) can be packed into a single message!
- We demand that **all nodes** know the MST at the end of the computation!
- How can the MST be constructed now?
- Let's look at a simple algorithm first...

I. Introduction: A Simple Algorithm

- All well-known MST algorithms (local or distributed) are based on the following lemma:

> **Lemma 1**: It is always safe to add an edge to the spanning tree, if this edge is the MWOE of a node v.



I. Introduction: A Simple Algorithm

Phase k: Code for node v in cluster F

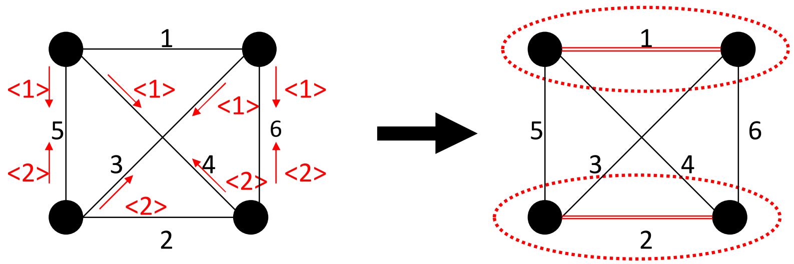
Input: Set of chosen edges that build node clusters

1. Compute the MWOE
2. Send the MWOE to all nodes in the same cluster
3. Receive messages from the other nodes
4. If own MWOE is the lightest, then broadcast it to all other nodes and add this edge (→ All edges have to know all clusters after each round)
5. Receive other broadcast messages and add those edges as well

I. Introduction: A Simple Algorithm

• Example: <w({v,u})> = <v,u,w({v,u})>

Round 1:



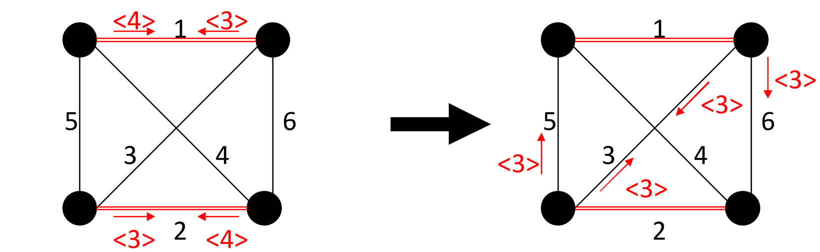
Broadcast the lightest edge to the other nodes

Add edges and update clusters

I. Introduction: A Simple Algorithm

• Example: <w({v,u})> = <v,u,w({v,u})>

Round 2:



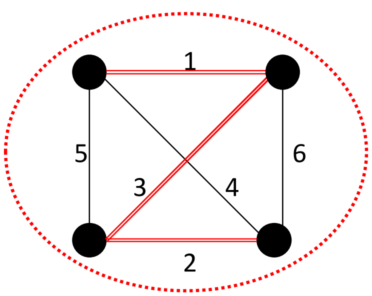
Send MWOE to all nodes in the same cluster

Broadcast the lightest edge to the other nodes

I. Introduction: A Simple Algorithm

• Example: <w({v,u})> = <v,u,w({v,u})>

Round 2:



Add edges and update clusters

The algorithm is obviously correct.
Since the minimum cluster size **doubles** in each round, the algorithm computes the MST in

O(log n) rounds!

Can it be improved?
Lower bound?

Overview

II. Related Work

- D: constant diameter of the graph (maximum distance between any two nodes of the graph).

	Known Algorithms	Known Lower Bounds
D = 1	$O(\log n)$	???
D = 2	$O(\log n)$	???
D ≥ 3	$O(D + \sqrt{n} \log^* n)$	$\Omega(n^{1/4} / \sqrt{\log n})$

II. Related Work

- Δ: constant diameter of the graph (maximum distance between any two nodes of the graph).

Even our simple algorithm has this complexity!!

Interesting "jump"!

	Known Algorithms	Known Lower Bounds
D = 1	$O(\log n)$	???
D = 2	$O(\log n)$	???
D ≥ 3	$O(D + \sqrt{n} \log^* n)$	$\Omega(n^{1/4} / \sqrt{\log n})$

II. Related Work

- We will now derive an algorithm with time complexity $O(\log \log n)$!

	Known Algorithms	Known Lower Bounds
D = 1	$O(\log \log n)$	???
D = 2	$O(\log n)$	???
D ≥ 3	$O(D + \sqrt{n} \log^* n)$	$\Omega(n^{1/4} / \sqrt{\log n})$

Overview

I. Introduction

II. Previous Results

III. Fast MST Algorithm

- General Idea & Problems
- The Algorithm Step by Step

IV. Analysis

V. Summary

VI. Extensive Example

III. Fast MST Algorithm: General Idea

- In order to reduce the number of rounds, obviously clusters have to **grow faster**!
- In our simple algorithm, we used the MWOE of each cluster to merge clusters.
- With this approach, the minimum cluster size **doubled** in each phase.
- It would certainly be faster if the k lightest outgoing edges of each cluster were used, where k is the number of nodes in the cluster!

→ This is exactly what our new algorithm will do!

III. Fast MST Algorithm: General Idea

- In order to reduce the number of rounds, obviously clusters have to **grow faster**!
- Let β_k denote the minimum cluster size after phase k, then it holds for our simple algorithms that

$$\beta_{k+1} \geq 2 \cdot \beta_k$$

and

$$\beta_0 := 1$$

thus

$$\beta_k \geq 2^k \rightarrow k \in O(\log n)$$

III. Fast MST Algorithm: General Idea

- Our goal is to improve the following inequality:

$$\beta_{k+1} \geq 2 \cdot \beta_k$$

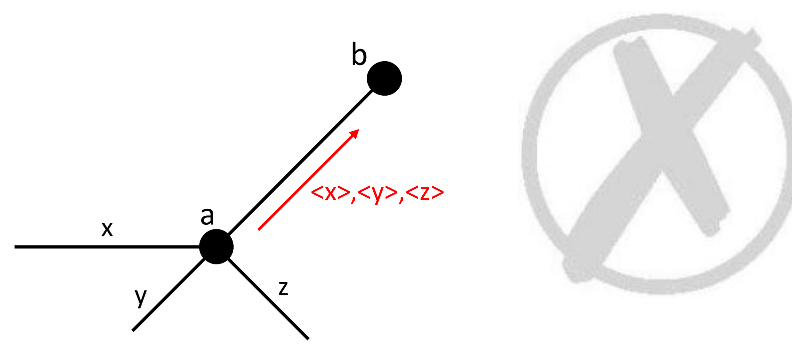
- We will derive an algorithm for which it holds that:

$$\beta_{k+1} \geq \beta_k \cdot (\beta_k + 1)$$

- Thus the cluster sizes grow quadratically as opposed to merely double in each phase! In order to achieve such a rate, information has to be spread faster!
- We will use a simple trick for that...

III. Fast MST Algorithm: General Idea

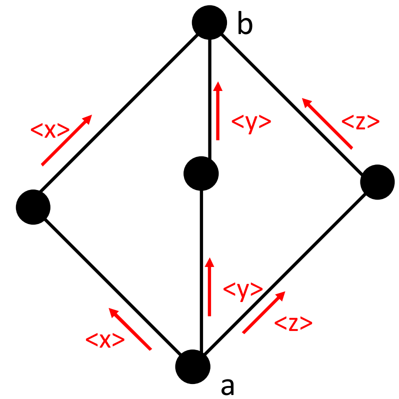
- Unfortunately, we cannot send a lot of information over a single link...



III. Fast MST Algorithm: General Idea

- However, we can send a lot of information from different nodes to a particular node v_0!
- A node can simply send parts of the information that it wants to transmit to a specific node to some other nodes. These nodes can send all parts to the specific node in one step!!
- This can be used to **share workload**!!!

III. Fast MST Algorithm: General Idea



We will use this trick twice in our algorithm!

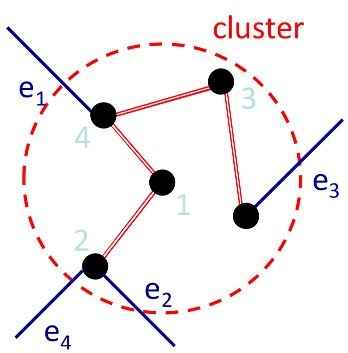
III. Fast MST Algorithm: General Idea

- Our new algorithm will execute the following steps in each phase.
- Let β be the minimal cluster size.

$\beta = 4$

1. Each cluster computes the β lightest edges $e_1,\ldots,e_\beta$ to other distinct clusters

2. Assign at most one of those lightest edges to the members of the cluster!



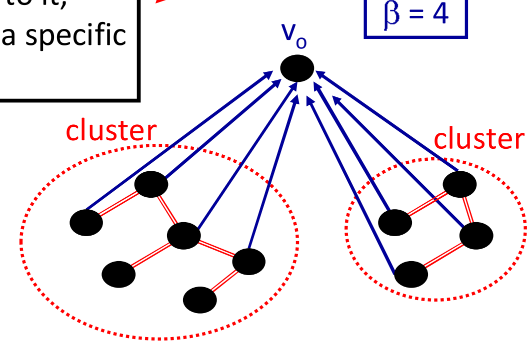
III. Fast MST Algorithm: General Idea

Step 2 and 3 together is exactly our trick!

3. Each node with an edge <v,u,w({v,u})> assigned to it, sends <v,u,w({v,u})> to a specific node v_0

4. Node v_0 computes the lightest edges that can be safely added to the spanning tree

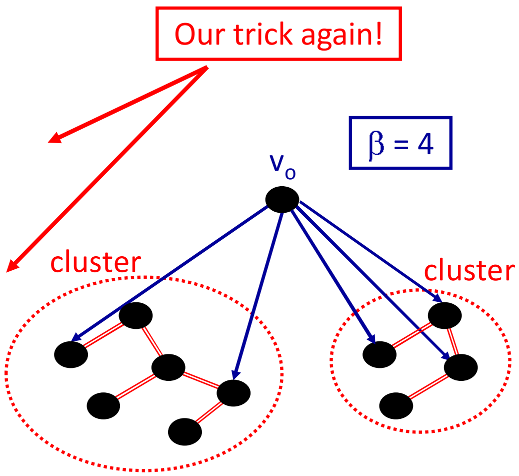
$\beta = 4$



III. Fast MST Algorithm: General Idea

5. Node v_0 sends a message to a node, if its assigned edge is added to the spanning tree

6. Each node, that received a message, broadcasts it to all other nodes (→ All nodes have to know about all added edges!)



III. Fast MST Algorithm: General Idea

- This way, more edges can be added in one phase!
- However, it is not clear yet how fast it really is...
- Furthermore, we do not know yet how these steps work in detail!!!
- There are a few obvious problems...

III. Fast MST Algorithm: Problems

- First problem:
- How can the β lightest outgoing edges of a specific cluster be computed?

→ This is actually not so difficult. The procedure Cheap_Out in the algorithm deals with this problem. We will discuss it in the following section.

III. Fast MST Algorithm: Problems

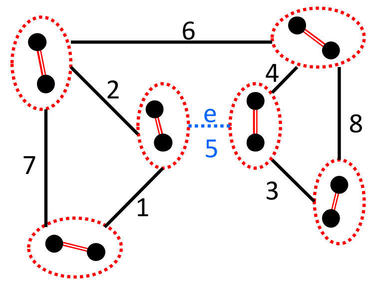
- Second problem:
- How can the designated node v_0 know which edges can be added **safely**?
- Let's illustrate this problem with an example graph!

III. Fast MST Algorithm: Problems

• In our example:

$|V| = n = 12$

$\beta = 2$ (minimum cluster size)

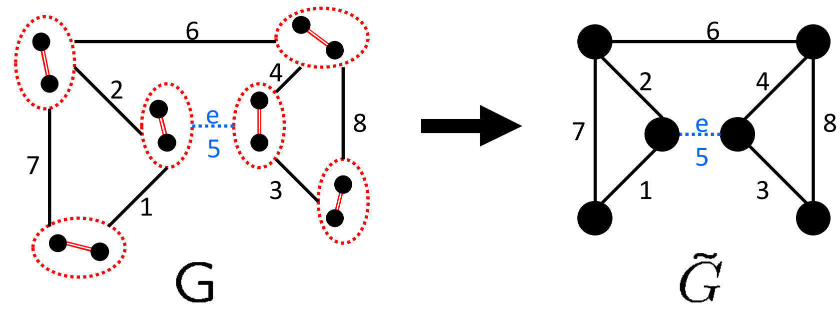


• This is the picture of the designated node v_0 after receiving the $\beta = 2$ lightest outgoing edges of each cluster

• v_0 does not know about the edge e! It is the 3rd lightest edge of both adjacent nodes!

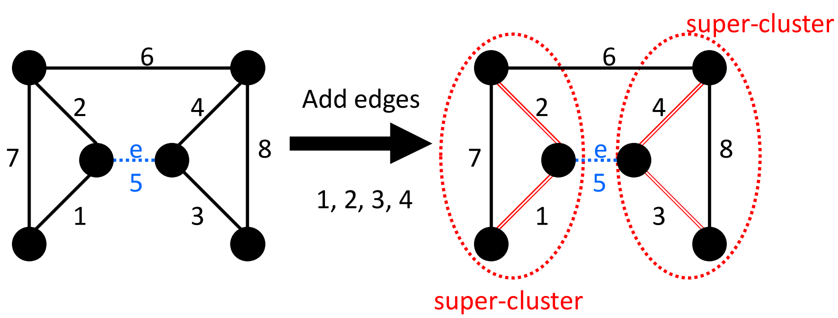
III. Fast MST Algorithm: Problems

• In our v_0 can construct a logical graph. Its nodes are the clusters and its edges are the $\beta = 2$ lightest outgoing edges.



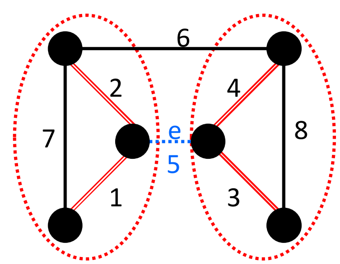
III. Fast MST Algorithm: Problems

• Based on the knowledge of the $\beta = 2$ lightest outgoing edges, v_0 can locally merge nodes of the logical graph into clusters. The 4 edges with weights 1, 2, 3 and 4 can be chosen safely, since always the MWOE is used.



III. Fast MST Algorithm: Problems

• If the edge with weight 6 is used to finish the construction of the spanning tree, then the resulting tree is not the MST!!!



The problem is that in **both** (super-)clusters at least one of the nodes has already used up all of its β outgoing edges. The $(\beta+1)$th outgoing edge might be lighter than other edges!!!

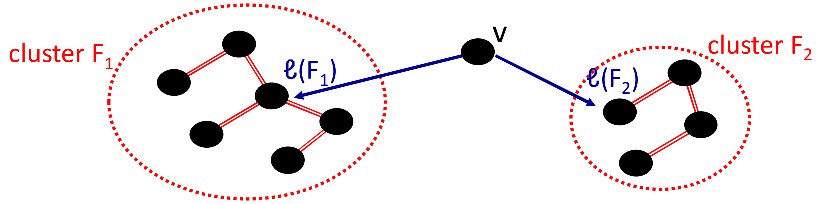
So, when is it safe to add an edge???

- Let's put everything together and solve the open problems!
- Initially, each node is itself a cluster of size 1 and no edges are selected.
- The algorithm consists of 6 steps. Each step can be performed in constant time.
- All 6 steps together build one phase of the algorithm, thus the time complexity of one phase is O(1).
- A specific node in each cluster F, e.g. the node with the smallest ID, is considered the leader ℓ(F) of the cluster F.

- Step 1

a) Each node v computes the minimum-weight edge e(v,F) that connects v to any node of cluster F for all clusters other than the own cluster

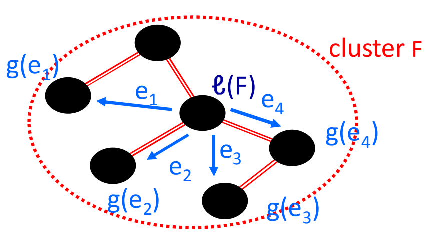
b) Each node v sends e(v,F) to the leader ℓ(F) for all clusters other than the own cluster



- Step 2

a) Each leader v of a cluster F (i.e. ℓ(F) =v) computes the lightest edge between F and every other cluster

b) Each leader v performs procedure Cheap_Out → Selects the β lightest outgoing edges and appoints them to its nodes



If edge e is appointed to v, then v is denoted e's guardian g(e)

- Procedure Cheap_Out

Code for the leader of cluster F

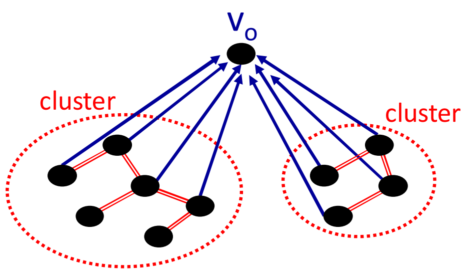
Input: Lightest edge e(F,F') for every other cluster F'

1. Sort the input edges in increasing order of weight
2. Define β = min{|F|, <# of clusters>}
3. Choose the first β nodes of the sorted list
4. Appoint the node with the ith largest ID as the guardian of the ith edge, i = 1,...,β
5. Send a message about the edge to the node it is appointed to

III. Fast MST Algorithm: The Algorithm Step by Step

- Step 3

All nodes that are guardians for a specific edge send a message to the designated node v_0, e.g. the node with the smallest ID in the graph



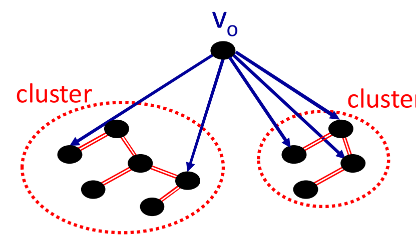
v_0 knows the β lightest outgoing edges of each cluster!

III. Fast MST Algorithm: The Algorithm Step by Step

- Step 4

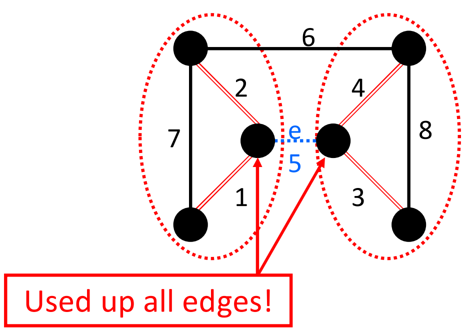
a) v_0 locally performs procedure Const_Frags → Computes the edges to be added

b) For all added edges, v_0 sends a message to g(e)



III. Fast MST Algorithm: The Algorithm Step by Step

- How does Const_Frags work?
- As we have seen before, a problem occurs when all β outgoing edges of a cluster are used up!
- More precisely, a problem occurs **only if** there is at least one cluster **in each** of the two super-clusters that are supposed to be merged!!

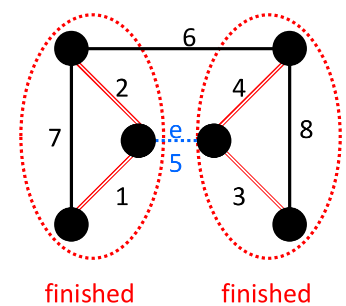


III. Fast MST Algorithm: The Algorithm Step by Step

- How does Const_Frags work?

Finished = not safe in the script

- We call a super-cluster containing a cluster that used up all of its β edges finished.
- If an edge is the lightest outgoing edge of one super-cluster that is **not finished**, then it is still safe to add it, no matter if the other super-cluster is finished, since we are sure that there is no better edge to connect the unfinished super-cluster to other clusters.



III. Fast MST Algorithm: The Algorithm Step by Step

- Procedure Const_Frags

Code for the designated node v_0

Input: the β lightest outgoing edges of each cluster

1. Construct the logical graph
2. Sort the input edges in increasing order of weight
3. Go through the list, starting with the lightest edge:

```
If the edge can be added without creating a cycle then
    add it
else
    drop it
```


III. Fast MST Algorithm: The Algorithm Step by Step

- Procedure Const_Frags

If two (super-)clusters are merged, then the new super-cluster is declared finished if

  - the edge is the heaviest edge of a cluster in any of the two super-clusters or
  - any of the two super-clusters is already finished.

- If the edge is dropped (→ both clusters already belong to the same super-cluster), then the super-cluster is declared finished if
  - the edge is the heaviest edge of any of the two clusters.

III. Fast MST Algorithm: The Algorithm Step by Step

- Procedure Const_Frags

Note: If a super-cluster is declared finished then it will remain finished until the end of the phase.

- Final Step

All edges between finished super-clusters are **deleted** (before looking at the next lightest edge)

Those are the dangerous edges!

III. Fast MST Algorithm: The Algorithm Step by Step

- Step 5

All nodes that received a message from v_0 broadcast their edge to all other nodes.

- Step 6

Each node adds all edges and computes the new clusters.

If the number of clusters is greater than 1, then the next phase starts.

III. Fast MST Algorithm: The Algorithm Step by Step

- The entire algorithm for node v in cluster F

1. Compute the minimum-weight edge e(v,F') that connects v to cluster F' and send it to ℓ(F') for all clusters F' ≠ F
2. if v = ℓ(F): Compute lightest edge between F and every other cluster. Perform Cheap_Out
3. if v = g(e) for some edge e: Send <e> to v_0
4. if v = v_0: Perform Const_Frags. Send message to g(e) for each added edge e
5. if v received a message from v_0: Broadcast it
6. Add all received edges and compute the new clusters

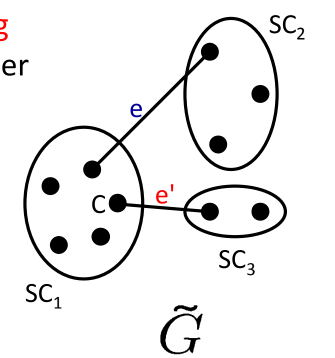
Overview

IV. Analysis: Correctness

- It suffices to show that whenever an edge is added, it is part of the MST → We only have to analyze Const_Frags!
- Proof [Sketch]: We only have to show that we always add the lightest outgoing edge of each super-cluster. Because of Lemma 1, this is always the right choice!

IV. Analysis: Correctness

- Assume edge e is used to merge super-cluster SC_1 and SC_2. W.l.o.g., assume that SC_1 is not finished and that e is one of the β lightest outgoing edges of its cluster.
- We will show now that e is the MWOE of SC_1!
- Assume that there is a lighter outgoing edge e' (w(e') < w(e)), incident to a cluster C that connects super-cluster SC_1 to super-cluster SC_3.



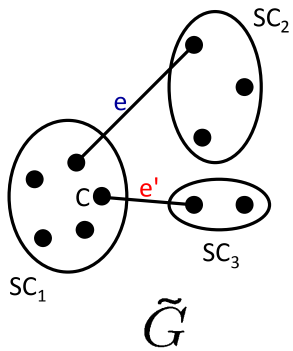
● is a cluster

IV. Analysis: Correctness

• Case 1: e' is among the β lightest outgoing edges of its cluster C.

→ Since w(e') < w(e), e' must have been considered before e, thus either SC_1 and SC_3 have been merged before or e' was dropped because $SC_1 = SC_3$. Either way, e' cannot be an outgoing edge when the algorithm adds e.

→ Contradiction!



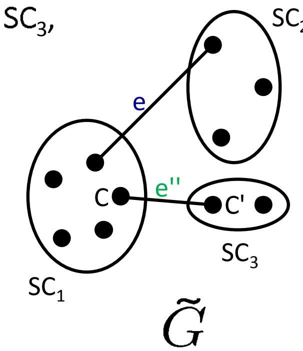
IV. Analysis: Correctness

• Case 2: e' is **not** among the β lightest outgoing edges of its cluster C.

• Case 2.1: There is an edge e'' among the β lightest outgoing edges from cluster C leading to the same cluster C'.

It follows that w(e'') < w(e'). Since $SC_1 \neq SC_3$, e'' has not been considered yet, thus w(e) < w(e'').
Hence we have that w(e) < w(e').

→ Contradiction!



IV. Analysis: Correctness

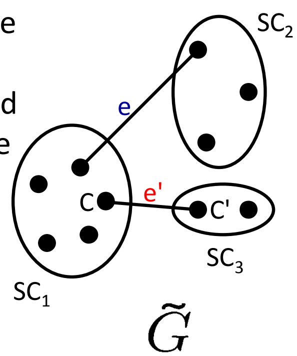
• Case 2: e' is **not** among the β lightest outgoing edges of its cluster C.

• Case 2.2: None of the β lightest outgoing edges of C lead to C'.

Thus, all β outgoing edges have lower weights than e', also the heaviest of these edges e'', i.e., w(e'') < w(e') < w(e).
Hence, edge e'' must have been inspected already. Since it is the heaviest (last) edge of some cluster, SC_1 must now be finished.

→ Contradiction!

■



IV. Analysis: Time Complexity

• Each phase requires O(1) rounds, but how many phases are required until termination?

•Reminder: β_k denotes the minimum cluster size in phase k.

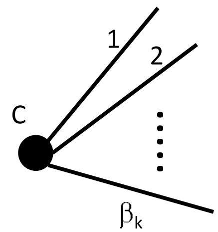
Lemma 2: It holds that

$$\beta_{k+1} \geq \beta_k(\beta_k+1).$$

IV. Analysis: Time Complexity

• Proof [Sketch]:
We prove a stronger claim: Whenever a super-cluster is declared finished in phase k+1, it contains at least β_k+1 clusters.

• Each cluster has (at least) β_k outgoing edges in phase k+1, since β_k is the minimum cluster size after phase k.

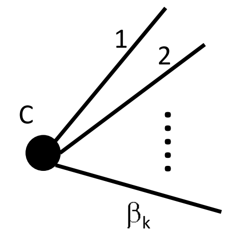


IV. Analysis: Time Complexity

• Case 1: The super-cluster is declared finished after one of its clusters has used up all of its β_k outgoing edges. Let C be this cluster.

• Let's call those edges 1, 2, ..., β_k leading to the clusters C_1, C_2, ..., C_β.

• If the inspection of an edge **does not** result in a merge, then the clusters already belong to the same super-cluster! If there is a merge, then they belong to the same super-cluster afterwards.

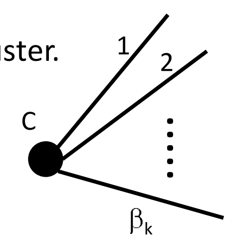


IV. Analysis: Time Complexity

• Thus, at the end, the super-cluster contains at least C, C_1, C_2, ..., C_β!
→ The super-cluster contains at least β_k+1 clusters.

• Case 2: The super-cluster is declared finished after merging with an already finished super-cluster.

• Using an inductive argument, the finished super-cluster must already contain at least β_k+1 clusters, since one of its clusters has used up all of its β_k edges. ■



IV. Analysis: Time Complexity

$\boxed{\textbf{Theorem 1}\text{: The time complexity is } O(\log\log n) \text{ rounds.}}$

• Proof: According to Lemma 2, it holds that $\beta_{k+1} \geq \beta_k(\beta_k+1)$. Furthermore, we have that $\beta_0 := 1$. Hence it follows that

$$\beta_k \geq 2^{2^{k-1}}$$

for every $k \geq 1$. Since $\beta_k \leq n$, it follows that $k \leq \log(\log n)+1$. Since each phase requires O(1) rounds, the time complexity is O(log log n). ■

IV. Analysis: Time Complexity

Theorem 2: The message complexity is $O(n^2 \log n)$.

Number of **bits**!

• The proof is simple: Count the messages exchanged in steps 1, 3, 4, and 5. We will not do this here.

• Adler et al. showed that the minimum number of bits required to solve the MST problem in this model is $\Omega(n^2 \log n)$. Thus, this algorithm is asymptotically optimal!

Overview

V. Summary: Results

• The presented algorithm solves the MST problem in the all-to-all communication model in $O(\log \log n)$ rounds.

• The algorithm sends $O(n^2 \log n)$ bits in total, which is asymptotically optimal.

V. Summary: Conclusions

"An obvious question we leave open is whether the algorithm can be improved, or whether there is an inherent lower bound of $\Omega(\log \log n)$ on the number of communication rounds required to construct an MST in this model."

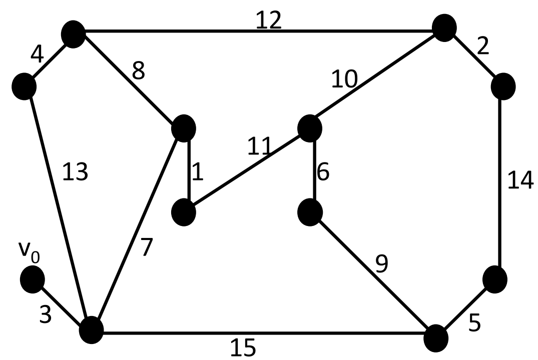
• Is there a faster algorithm?

• Is $\Omega(\log \log n)$ a lower bound?

• Is there an algorithm with time complexity $O(\log \log n)$ for graphs of diameter 2?

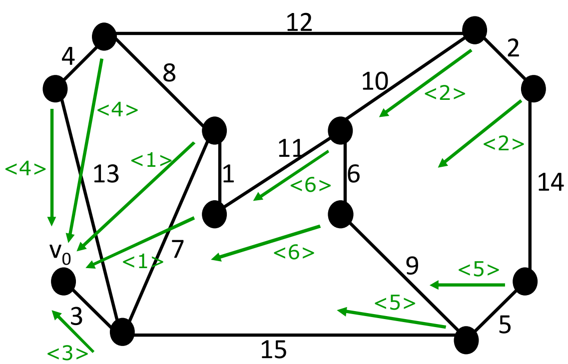
Overview

VI. Extensive Example: The Graph



All other edges are heavier!!!

VI. Extensive Example: Phase 1

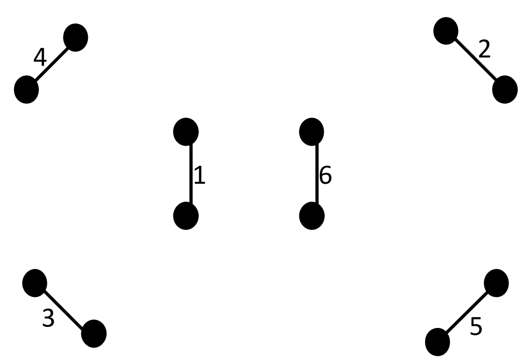


1. Not necessary
2. Not necessary
3. Send MWOE to v_0
4. Const_Frags!

VI. Extensive Example: Phase 1

- Const_Frags

1. Construct logical graph

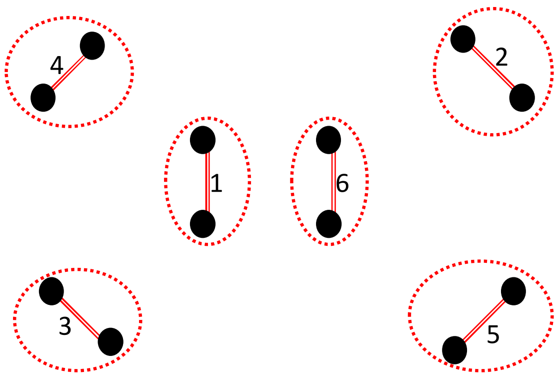


$\tilde{G}$

VI. Extensive Example: Phase 1

- Const_Frags

2. Add edges



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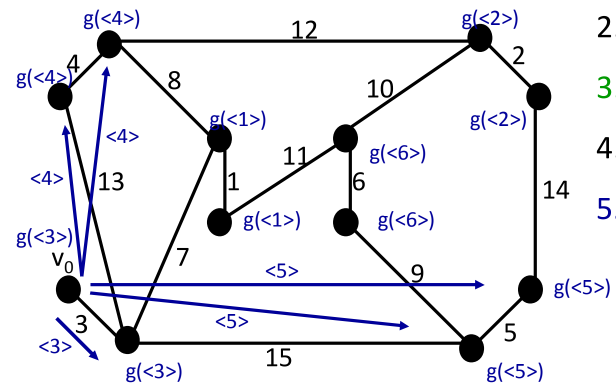
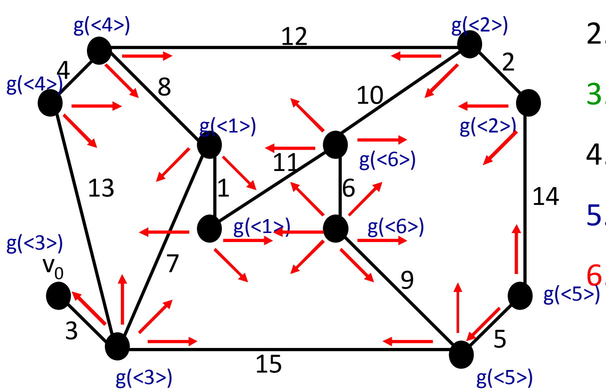
VI. Extensive Example: Phase 1



1. Not necessary
2. Not necessary
3. Send MWOE to v_0
4. Const_Frags!
5. Send e to g(e)

Only some messages are displayed

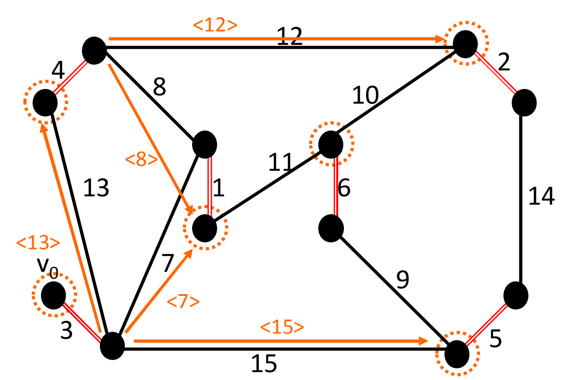
VI. Extensive Example: Phase 1



1. Not necessary
2. Not necessary
3. Send MWOE to v_0
4. Const_Frags!
5. Send e to g(e)
6. Broadcast e and update the clusters

Only some messages are displayed

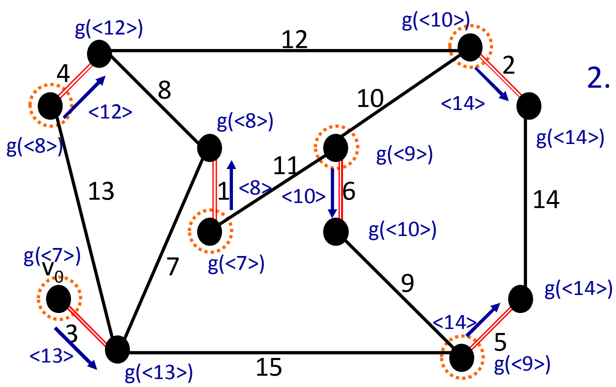
VI. Extensive Example: Phase 2



1. Compute e(v,F‘) and send it to ℓ(F‘)

Only some messages are displayed

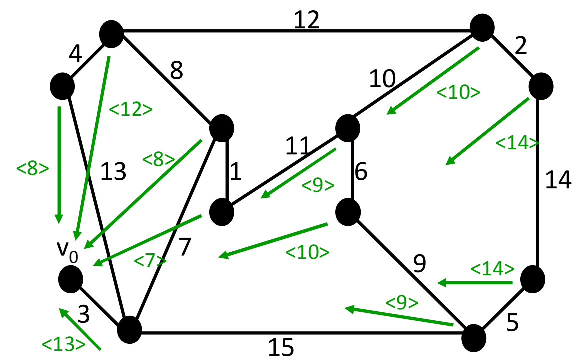
VI. Extensive Example: Phase 2



Only some messages are displayed

1. Compute e(v,F‘) and send it to ℓ(F‘)
2. Select β = 2 lightest outgoing edges and appoint guardians

VI. Extensive Example: Phase 2

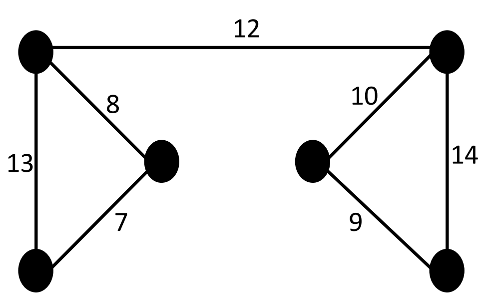


1. Compute e(v,F‘) and send it to ℓ(F‘)
2. Select β = 2 lightest outgoing edges and appoint guardians
3. Send appointed edge to v_0
4. Const_Frags!

VI. Extensive Example: Phase 2

- Const_Frags

1. Construct logical graph

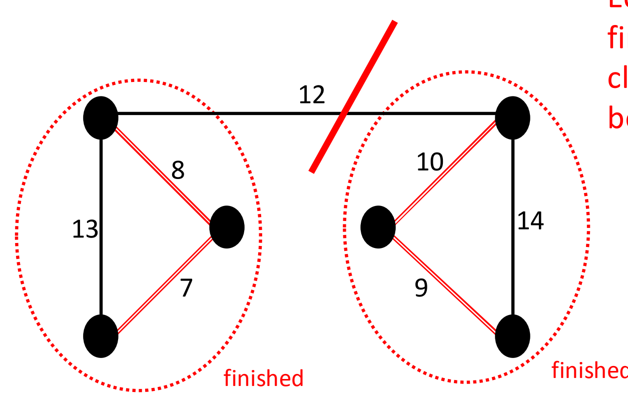


$\tilde{G}$

VI. Extensive Example: Phase 2

- Const_Frags

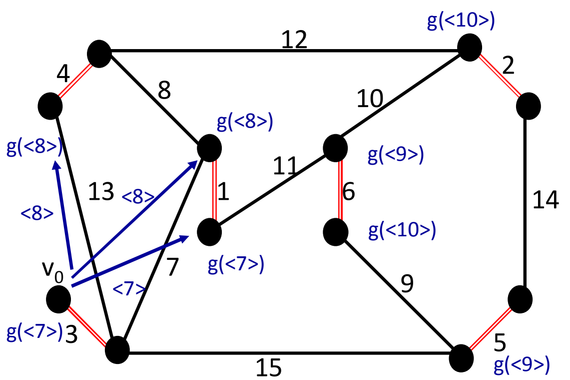
2. Add edges



Edges between finished super-clusters must not be added!

$\tilde{G}$

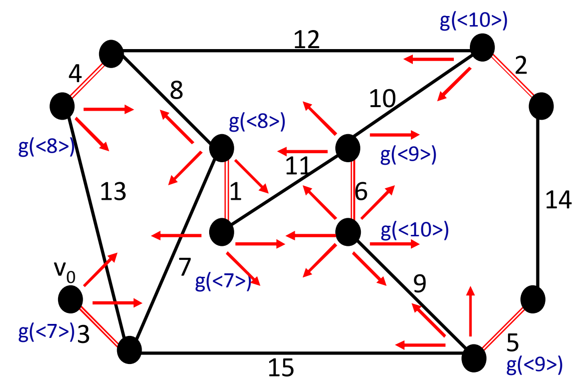
VI. Extensive Example: Phase 2



1. Compute e(v,F') and send it to ℓ(F')
2. Select β = 2 lightest outgoing edges and appoint guardians
3. Send appointed edge to v_0
4. Const_Frags!
5. Send e to g(e)

Only some messages are displayed

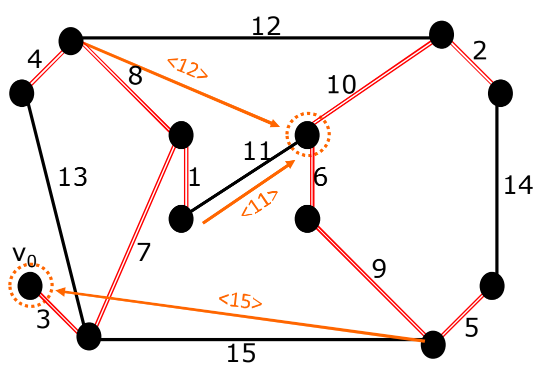
VI. Extensive Example: Phase 2



1. Compute e(v,F') and send it to ℓ(F')
2. Select β = 2 lightest outgoing edges and appoint guardians
3. Send appointed edge to v_0
4. Const_Frags!
5. Send e to g(e)
6. Broadcast e and update the clusters

Only some messages are displayed

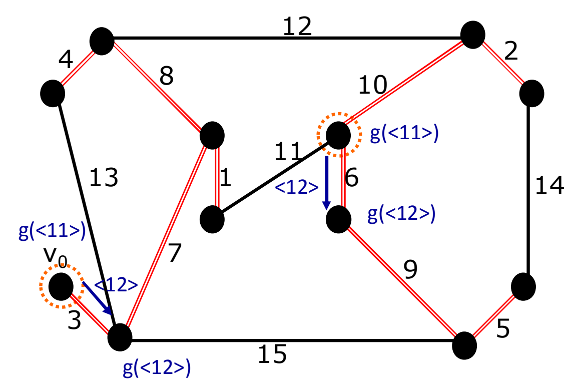
VI. Extensive Example: Phase 3



1. Compute e(v,F') and send it to ℓ(F')

Only some messages are displayed

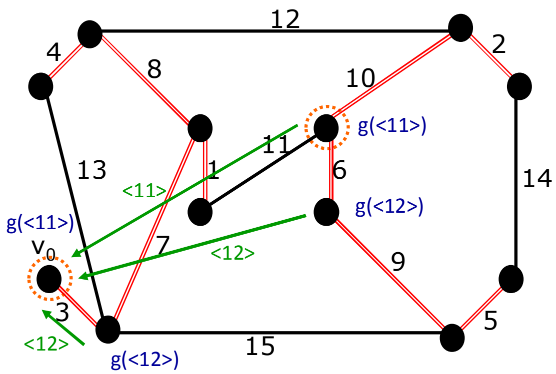
VI. Extensive Example: Phase 3



1. Compute e(v,F') and send it to ℓ(F')
2. Select β = 2 lightest outgoing edges and appoint guardians

Only some messages are displayed

VI. Extensive Example: Phase 3



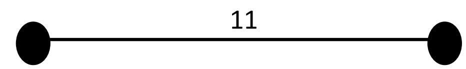
Only some messages are displayed

1. Compute e(v,F‘) and send it to ℓ(F‘)
2. Select β = 2 lightest outgoing edges and appoint guardians
3. Send appointed edge to v_0
4. Const_Frags!

VI. Extensive Example: Phase 3

- Const_Frags

1. Construct logical graph

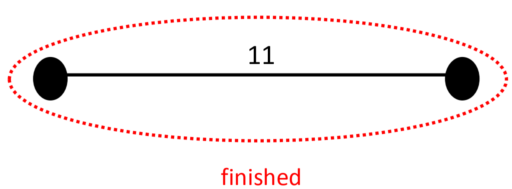


$\tilde{G}$

VI. Extensive Example: Phase 3

- Const_Frags

2. Add edges



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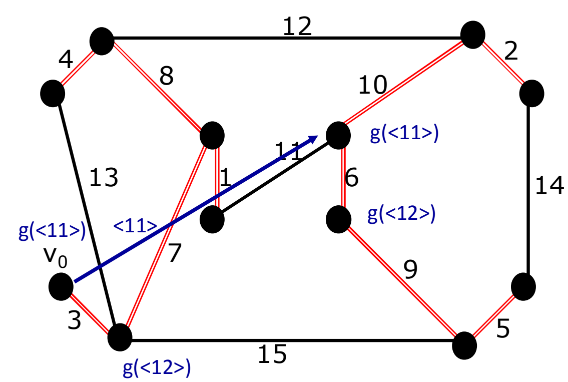
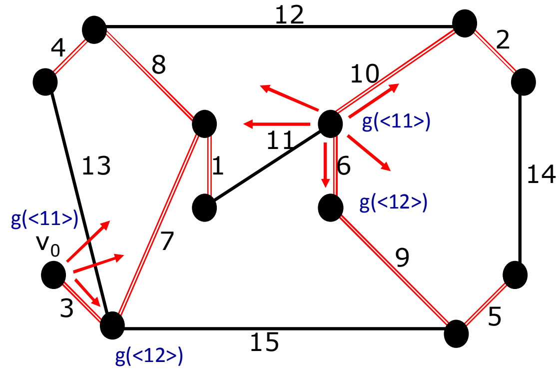
VI. Extensive Example: Phase 3



1. Compute e(v,F‘) and send it to ℓ(F‘)
2. Select β = 2 lightest outgoing edges and appoint guardians
3. Send appointed edge to v_0
4. Const_Frags!
5. Send e to g(e)

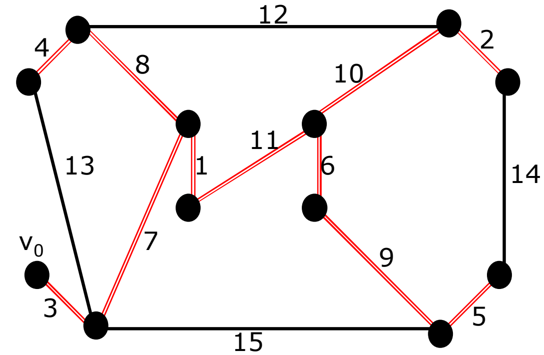
VI. Extensive Example: Phase 3



Only some messages are displayed

1. Compute e(v,F') and send it to ℓ(F')
2. Select β = 2 lightest outgoing edges and appoint guardians
3. Send appointed edge to v_0
4. Const_Frags!
5. Send e to g(e)
6. Broadcast e and update the clusters

VI. Extensive Example: After Phase 3



Done!

References

• M. Adler, W. Dittrich, B. Juurlink, M. Kutylowski, and I. Rieping. Communication Optimal Parallel Minimum Spanning Tree Algorithms. In Proc. 10th Annual ACM Symposium on Parallel Algorithms and Architectures (SPAA), pages 27-36, 1998.

• Z. Lotker, B. Patt Shamir, and D. Peleg. Distributed MST for Constant Diameter Graphs. In Proc. 20th Annual ACM Symposium on Principles of Distributed Computing (PODC), pages 63-71, 2001.

• Z. Lotker, E. Pavlov, B. Patt Shamir, and D. Peleg. MST Construction in O(log log n) Communication Rounds. In Proc. 15th Annual ACM Symposium on Parallel Algorithms and Architectures (SPAA), pages 94-100, 2003.

• D. Peleg and V. Rubinovich. Near Tight Lower Bound on the Time Complexity of Distributed MST Construction. SIAM J. Comput., 30:1427-1442, 2000.